KIT Department of Informatics
Institute for Anthropomatics and Robotics (IAR)
High Performance Humanoid Technologies (H^2T)

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Solution Sheet 6

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Solution 1 (Color Representation)

1. RGB: (120, 80, 210)

   Hue:

$$
\begin{aligned}
c &= cos^{-1}\left(\frac{2R-G-B}{2\sqrt{(R-G)^2+(R-B)(G-B)}}\right) \\
&= cos^{-1}\left(\frac{240-80-210}{2\sqrt{(120-80)^2+(120-210)(80-210)}}\right) \\
&= cos^{-1}\left(\frac{-50}{2\sqrt{1600+11700}}\right) \\
&\approx cos^{-1}(-0.2168) \\
&\approx 102.5^\circ \\
H &= 360^\circ - c \approx 257.5^\circ \text{ , as } B \geq G
\end{aligned}
$$

   Saturation:

$$
\begin{aligned}
S &= 1 - \frac{3}{R+G+B} min(R,G,B) \\
&= 1 - \frac{3}{120+80+210} 80 \\
&= 1 - \frac{240}{410} \\
&\approx 0.415
\end{aligned}
$$

   Intensity:

$$
\begin{aligned}
I &= \frac{R+G+B}{3} \\
&= \frac{410}{3} \\
&\approx 136.7
\end{aligned}
$$

2. RGB: (0, 150, 130)

   Hue:

$$\begin{aligned}\theta &= cos^{-1}\left(\frac{2R-G-B}{2\sqrt{(R-G)^2+(R-B)(G-B)}}\right)\\ &= cos^{-1}\left(\frac{0-150-130}{2\sqrt{(0-150)^2+(0-130)(150-130)}}\right)\\ &= cos^{-1}\left(\frac{-280}{2\sqrt{22500-2600}}\right)\\ &\approx cos^{-1}(-0.9924)\\ &\approx 172.9^\circ\\ H = \theta &\approx 172.9^\circ \text{ , as } B < G\end{aligned}$$

   Saturation:

$$\begin{aligned}S &= 1 - \frac{3}{R+G+B}min(R,G,B)\\ &= 1 - 0 = 1\end{aligned}$$

   Intensity:

$$\begin{aligned}I &= \frac{R+G+B}{3}\\ &= \frac{280}{3}\\ &\approx 93.3\end{aligned}$$

3. **Dimming a laboratory light:** R, G and B change uniformly, i.e. increase or decrease by the same factor. H and S remain unchanged, I changes according to the change in brightness.

4. **Natural light changes from sunny to cloudy:** As not only the light intensity changes here, but also the color spectrum of the light, all values change, including H and S - although in most cases significantly less than the RGB values.

Solution 2 (Camera Modell)

1.

$$\begin{aligned}u &= \frac{f}{z}x\\ x &= u\frac{z}{f}\\ x &= 0,8mm\frac{350m}{20mm}\\ x &= 0,8\frac{350m}{20}\\ x &= 0,4 \cdot 35m = 14m\end{aligned}$$

2.

$$100\frac{\pi}{2}m \approx 157m$$
$$314px = \frac{20mm}{800m}157m$$
$$314px = \frac{20mm}{800}157$$
$$2px = \frac{20mm}{800}$$
$$2px = \frac{1mm}{40}$$
$$80px = 1mm$$

Solution 3 (Filters in Image Processing)

1. The edge pixels are ignored, the remaining pixels are calculated as follows:

$$\begin{aligned} B_x(2,2) &= -1B(1,1) + 0B(1,2) + 1B(1,3) - 1(B(2,1) + 0B(2,2) + \\ &\quad 1B(2,3) - 1B(3,1) + 0B(3,2) + 1B(3,3) \\ &= -20 + 0 + 20 - 20 + 0 + 20 - 20 + 0 + 20 = 0 \\ B_x(2,3) &= -20 + 0 + 20 - 20 + 0 + 20 - 20 + 0 + 20 = 0 \\ B_x(2,4) &= -20 + 0 + 40 - 20 + 0 + 40 - 20 + 0 + 40 = 60 \end{aligned}$$

etc., the results is given by

$$B_x = \begin{pmatrix} 0 & 0 & 60 & 60 & 0 & 0 & -30 & -30 & 0 \\ 0 & 0 & 40 & 40 & 0 & 0 & -50 & -50 & 0 \\ 30 & 0 & 20 & 20 & 0 & 0 & -70 & -70 & 0 \\ 60 & 30 & 0 & 0 & 0 & 0 & -90 & -90 & 0 \\ 60 & 60 & 30 & 0 & 0 & 0 & -90 & -90 & 0 \end{pmatrix}$$

2.

$$B_y = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 90 & 90 & 70 & 50 & 30 & 30 & 10 & -10 & -30 \\ 60 & 90 & 70 & 50 & 30 & 30 & 10 & -10 & -30 \\ -60 & -30 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -60 & -60 & -30 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Solution 4 (Threshold Segmentation)

1.
$$Img'(u,v) = \begin{cases} 255 & Img(u,v) > T \\ 0 & \text{else} \end{cases} \tag{1}$$

- Threshold segmentation is applied with a threshold of $T = 51$
- Pixels with a values of $Img(u,v) > 51$ are set to 255, else to 0
- Resulting segmented image, see Figure 2

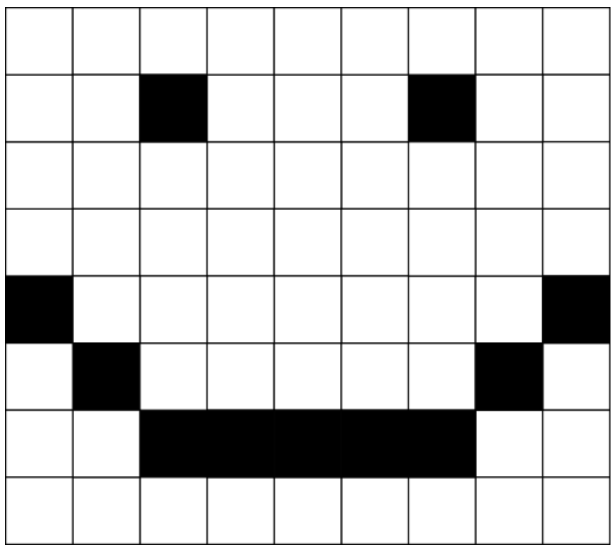

Figure 1: Segmented image.

2. Masking the image using the segmented image

255	212	74	181	176	176	196	171	156
181	219		135	163	56		69	56
186	148	79	186	230	163	64	54	84
166	237	69	179	166	69	120	112	245
	194	194	179	107	117	99	87	
222		186	163	115	77	105		71
207	186						54	201
189	196	89	64	128	71	89	74	54

Figure 2: Segmented image.

Solution 5 (Iterative Closest Point (ICP))

1. Error function F_T :

$$F_T(V) = \sum_{j=0}^{2} \min_i ||v_j - p_i||^2 \tag{2}$$

$$v_0 : p_0, \qquad v_1 : p_0, \qquad v_2 : p_0 \tag{3}$$

$$F_T(V) = ||v_0 - p_0||^2 + ||v_1 - p_0||^2 + ||v_2 - p_0||^2 \tag{4}$$

2. To solve the problem the ICP-algorithm is to be applied using the steepest gradient approach. Determine the simplified error function.

   - $F'_T = ||v_0 - p_0||^2$
   - Determine the gradient: $\nabla F'_T$

$$F'_T = \left(\sqrt{(v_{0,x} - p_{0,x})^2 + (v_{0,y} - p_{0,y})^2}\right)^2 \tag{5}$$

$$\nabla F'_T = 2\begin{pmatrix} v_{0,x} - p_{0,x} \\ v_{0,y} - p_{0,y} \end{pmatrix} = 2(v_0 - p_0) \tag{6}$$

3. For the ICP algorithm from the second part of the task, specify the function that approximates V to P with a step size α. Draw the first two iterations for $\alpha = 0.25$ in Figure 3.

$$V_0 = \{(0,0)^T, (1,0)^T, (0,1)^T\} \tag{7}$$

$$v_{0,1} = v_{0,0} - \alpha \nabla F'_T = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 0.25 \cdot 2 \cdot \begin{pmatrix} 0-2 \\ 0-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{8}$$

$$V_1 = \{(0,0)^T, (1,0)^T, (0,1)^T\} \tag{9}$$

$$v_{0,1} = v_{0,0} - \alpha \nabla F'_T = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 0.25 \cdot 2 \cdot \begin{pmatrix} 1-2 \\ 1-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{10}$$

$$V_2 = \{(1.5,1.5)^T, (2.5,1.5)^T, (1.5,2.5)^T\} \tag{11}$$

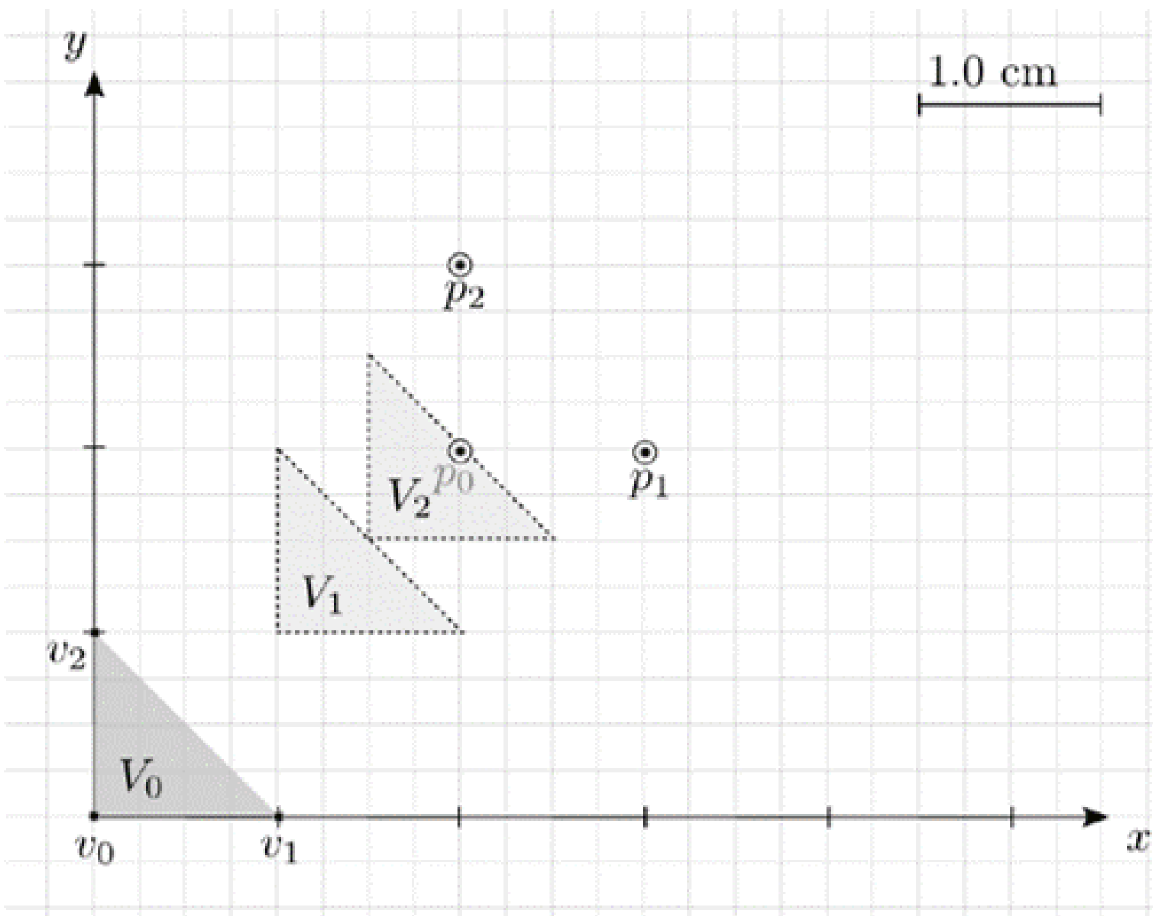

Figure 3: First 2 steps of ICP.